Imprints of superfluidity on magneto-elastic QPOs of SGRs

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Our numerical simulations show that axisymmetric, torsional, magneto-elastic oscillations of magnetars with a superfluid core can explain the whole range of observed quasi-periodic oscillations (QPOs) in the giant flares of soft gamma-ray repeaters. There exist constant phase, magneto-elastic QPOs at both low ($f < 150$Hz) and high frequencies ($f > 500$Hz), in full agreement with observations. The range of magnetic field strengths required to match the observed QPO frequencies agrees with that from spin-down estimates. These results strongly suggest that neutrons in magnetar cores are superfluid.

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Neutron stars are perfect astrophysical laboratories to study the equation of state (EoS) of matter at supra-nuclear densities, i.e., at conditions impossible to replicate on Earth. Giant flares of Soft Gamma-ray Repeaters (SGRs) are very promising events that can be used to obtain information about the structure of neutron stars, since it is believed that their source are highly magnetized neutron stars (magnetars) [1] suffering a global rearrangement of the magnetic field, and possibly involving a fracture of the solid crust. In the X-ray light curves of two of the three giant flares detected so far, SGR 1806-20 and SGR 1900+14, a number of Quasi-Periodic Oscillations (QPOs) have been observed [2, 3]. This may have been the first detection of neutron star oscillations, which provide a possibility for studying such compact stars through asteroseismology. The observed frequencies consist of two categories, *low frequency QPOs* between a few tens of Hz and up to 150 Hz observed in both events, and *high frequency QPOs* above 500 Hz, which are only observed in the 2004 giant flare. Some QPO frequencies roughly match those of discrete crustal shear modes in non-magnetized stars, namely $n = 0$ torsional modes (nodeless in the radial direction) for the low frequency QPOs and $n \geq 1$ modes for the high frequency QPOs (see [4–8] and references therein). However, these crustal modes are quickly damped by the magnetic field in the core [12–18].

On the other hand, torsional Alfvén oscillations (fundamental mode ~ 30 Hz), i.e. QPOs trapped at turning-points or edges of the Alfvén continuum of the highly magnetized core, can also have frequencies similar to those of the observed QPOs for magnetar field strengths of order $B \sim 10^{15}$ G, with the additional attractive feature of overtones appearing at near-integer ratios [9–11].

The Alfvén QPO model extended to magneto-elastic QPOs and different types of magnetic fields [13, 18] explains the observed low frequency QPOs as excitations of a fundamental turning-point QPO and of several overtones. However, the observation of high frequency QPOs poses a problem for this model, because the first overtone ($n = 1$) crustal shear mode is quickly absorbed into the Alfvén continuum [13, 17] and because there is no known mechanism to excite a specific high-order overtone of the turning-point magneto-elastic QPOs with the appropriate frequencies. A model explaining both low- and high-frequency QPOs would thus be a significant step towards a better understanding of neutron star interiors.

Previous models have considered a normal fluid (i.e., non-superfluid) consisting of neutrons, protons, and electrons in the core of the neutron star. This is a valid approach if the interaction between the different species is strong. However, theoretical calculations favor the presence of superfluid neutrons [19]. This idea is supported by the theory of pulsar glitches [20] and by the fact that the cooling curve of Cas A is consistent with a phase transition to superfluid neutrons [21, 22]. In this case the matter in the core of neutron stars cannot be described by a single-fluid approach. The effect of superfluidity in the oscillation spectrum of unmagnetized stars has been estimated in [23–28] and in the context of magnetars in [16, 17, 29–32]. The main consequence of a superfluid core is an increase in frequency of the Alfvén continuum bands by a factor of several with respect to the normal fluid, for the same magnetic field strength. It was suggested in [30] that such an increase (in conjunction with stratification) could account for the observed high frequency QPOs as fundamental, polar ($m = 2$) non-axisymmetric Alfvén modes, although this model cannot simultaneously accommodate the lowest observed frequency QPOs. How superfluid neutrons in the crust would affect the spectrum of shear oscillations was studied both for magnetized and unmagnetized models in [32–34].

Here, we investigate the effect of a superfluid core on the turning-point magneto-elastic QPOs of magnetars. Superfluidity is handled in our model by decoupling the superfluid neutrons in the core of the neutron star completely, i.e., we assume that there is no entrainment between neutrons and protons, and no direct interaction between both species. Hence, neutrons affect protons only through their gravitational interaction. Protons are expected to be superconducting in the core of neutron stars [19], but the magnetic field inside a magnetar may suppress superconductivity beyond a critical field strength that is estimated to be in the range 10^{15} G

$\lesssim B_{\rm core} \lesssim 10^{16}$ G [29]. Therefore, we consider normal (non-superconducting) protons in the core. In addition, since magnetars are slow rotators with periods of ~ 10 s, we neglect effects due to rotation that could create superfluid vortices.

The results presented here are obtained with the numerical code MCOCOA that solves the general-relativistic MHD equations [10, 35] including a treatment of elastic terms for the neutron star crust [13, 14, 18]. The influence of a superfluid phase of neutrons coexisting with a normal fluid can be described by the entrainment, a measure of the interaction of the different species. In the crust the interaction of the superfluid neutron component with the nuclei of the lattice due to Bragg reflection is so strong [36] that the perturbation of the lattice will carry along most of the superfluid neutrons. Therefore, we assume complete entrainment and treat the crust as if it was a single fluid with shear, including the total mass of all constituents. In the core we assume for simplicity that the neutrons are completely decoupled, i.e., only protons are dynamically linked to the magneto-elastic oscillations. This extreme approximation (complete decoupling) complements the one in our previous work, where we assumed complete coupling. The proton fraction in the core has been estimated to be $X_p \sim 0.05$ [38–42], and we have assumed this value in all our calculations (a more detailed treatment would consider a particular stratification). The dynamical behavior of electrons can be neglected because of their small mass.

For the evolution we have to solve the momentum and the induction equation. The latter remains unchanged compared to our previous work [10, 13], while the former one holds now for protons only. Effectively, we change the momentum of the fluid in the core in the φ-direction by replacing the total rest-mass density ρ by the rest-mass density of protons $\rho_p = X_p \rho$ only. The superfluid neutrons are not influenced by the torsional magneto-elastic oscillations.

Since the system under consideration consists of crust and core that have different properties, it is not obvious whether there exist discrete eigenmodes. Hence, to differentiate between discrete and continuum oscillations we use the phase of the Fourier transform of the time evolution. For discrete modes the whole star oscillates with the same phase. In contrast, the continuum of torsional Alfvén oscillations gives rise to a continuous phase change as one crosses field lines, because the eigenfrequencies of neighboring field lines are slightly different, i.e., the oscillation at these lines are out of phase leading to phase mixing and damping of the oscillations.

For a magnetar model with a normal fluid core, a crust, and a poloidal magnetic field there exist three distinct types of torsional magneto-elastic QPOs [13]: For weak surface magnetic fields, $B \lesssim 10^{15}$ G, the QPOs are reflected at the core-crust interface and the different field lines are weakly coupled through this boundary. At strong magnetic fields, $B > 5 \times 10^{15}$ G, the field dominates over the crustal shear modulus, i.e., the magneto-elastic QPOs reach the surface and individual field lines are coupled by the entire crust instead of only at the core-crust interface. For intermediate field strengths the magneto-

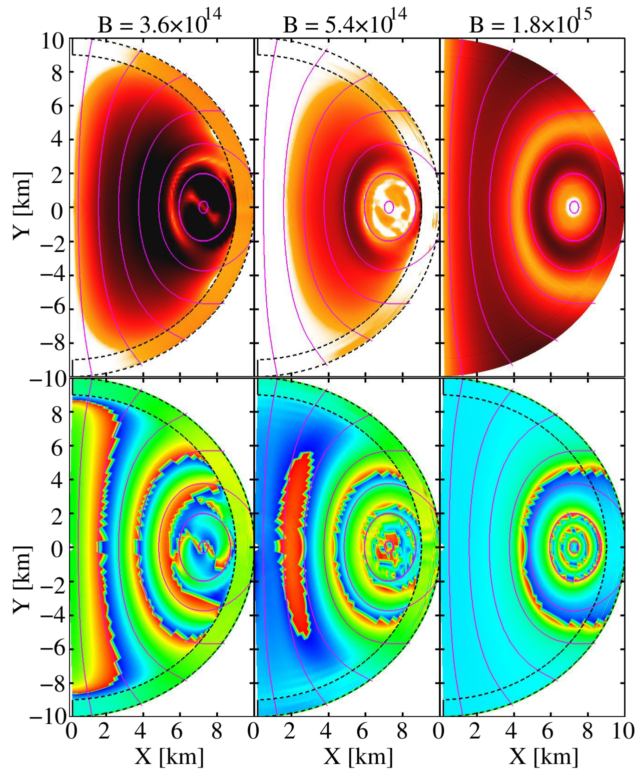

FIG. 1: Effective amplitude *top row* and phase *bottom row* of a particular QPO (see text for details) ranging from white-blue (minimum) to orange-red (maximum), and from $\theta = -\pi/2$ (blue) to $\theta = \pi/2$ (orange-red), respectively. The crust is indicated by the dashed black line, and magnetic field lines are given by the magenta lines. The bottom right panel demonstrates that for a typical magnetar surface field strength a discrete QPO with a constant phase (modulo numerical inaccuracies at a nodal line) exists in the whole open field line region. One can also recognize that the phase changes in regions with vanishing amplitudes only.

elastic QPOs change from being reflected at the core-crust interface to being reflected at the surface of the star. In all these cases no discrete modes exist.

If all superfluid neutrons in the core are decoupled we find that there are still no discrete modes for weak magnetic fields of $B \lesssim$ few $\times 10^{14}$ G. However, at typical magnetar surface field strengths of $B \sim 10^{15}$ G there exist QPOs with an almost constant phase in nearly the whole open field line region. This transition is exemplified in Fig. 1 for the lowest frequency QPO, but holds for other QPOs as well. The particular model shown in Fig. 1 has a mass of $1.4\ M_\odot$ and was computed with the APR EoS in the core [40] and the DH EoS in the crust [41]. For $B \gg 10^{15}$ G we expect the continuum to appear again, which is present in simulations without crust.

The different regimes can be explained with the speed of perturbations propagating along magnetic field lines, which exhibits a discontinuity at the core-crust interface. For weak magnetic fields $B \lesssim 10^{14}$ G, the shear speed is much higher than the Alfvén speed at the base of the crust, i.e., there is a large jump in the propagation speed at the crust-core interface.

This leads to a significant reflection of the QPOs at the core-crust interface, confining the QPOs mostly to the core (Fig.1, top left panel). In the superfluid case, the Alfvén speed in the core is a factor $\sqrt{1/X_p}$ higher than for a normal fluid, because $v_A^2 \sim B^2/\rho$ and only protons (a fraction X_p of the total mass) take part in the magneto-elastic oscillations. Hence, the jump in propagation speed at the crust-core interface is significantly smaller giving rise to less reflection and stronger penetration of the magneto-elastic oscillations into the crust. At $B \sim 10^{15}$ G the jump vanishes, and the Alfvén speed in the core approaches the shear speed at the base of the crust. The strong coupling of the magnetic field lines by the crust then leads to the appearance of oscillations with constant phase in the region of open field lines. Similar effects were observed in [12] and [10] for strong (numerical) viscosity.

The above effect is less pronounced in the normal fluid case, because there the transition from reflection at the core-crust interface to dominance of magnetic over shear effects in the crust occurs between 10^{15} G$< B < 5 \times 10^{15}$ G, while in the superfluid case the transition already starts at a few 10^{14} G. In addition, a more massive core takes part in the magneto-elastic oscillations in the normal fluid case, i.e., the coupling to the crust is weaker.

We now turn to the high frequency QPOs with $f > 500$ Hz, whose preferential excitation could not easily be justified in the magneto-elastic model with a normal fluid core. In a first attempt to include the effects of superfluidity, VanHoven & Levin [17] assumed that only 5% of the core takes part in the magneto-elastic oscillations. In their simulations, the $n = 1$ crustal shear modes are absorbed very efficiently into the core when initially only the crust is excited. In Fig. 2 we show the corresponding overlap integral (a measure for the excitation of a given crustal mode, see [13]) for a simulation with $B = 10^{15}$ G and with a $n = 1$, $l = 2$ crustal shear mode as initial perturbation. We obtain initial damping time scales of a few milliseconds for the superfluid and normal fluid cases, in broad agreement with [17]. However, the damping does not continue at the initial rate (see the inset in Fig. 2). After about 10 ms almost stable oscillations with modulating amplitudes persist at a similar frequency for both fluid models with much lower damping rates.

Fourier transforming the data for evolution times of about 1 s we find that the crustal $n = 1$ shear mode ($f \sim 760$ Hz) excites a global magneto-elastic QPO with $f \sim 893$ Hz in the superfluid case. For the normal fluid we find three magneto-elastic QPOs in the crust with $f \sim 782$, 806, and 829 Hz, respectively. In Fig. 3 we show the Fourier amplitude of the (azimuthal) velocity inside the crust close to the equator for the normal fluid and close to the pole for the superfluid model. The corresponding spatial structures of the strongest QPOs of both models are displayed in Fig. 4. In both cases the radial structure of the $n = 1$ QPO remains similar to that of the pure crustal shear mode inside the crust. However, its angular dependence differs considerably from that of the original spherical harmonic one due to the interaction with the core (see also [13] for the normal fluid case).

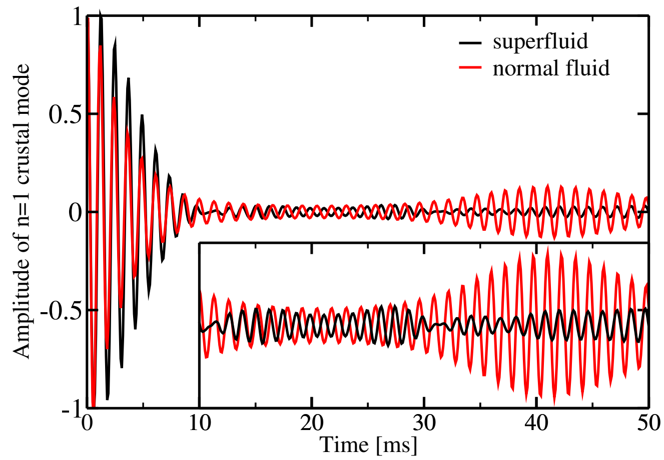

FIG. 2: Overlap integral with the $n = 1$ crustal shear mode at $B = 10^{15}$ G for normal and superfluid models. The inset shows a magnification of the amplitude from 10 to 50 ms.

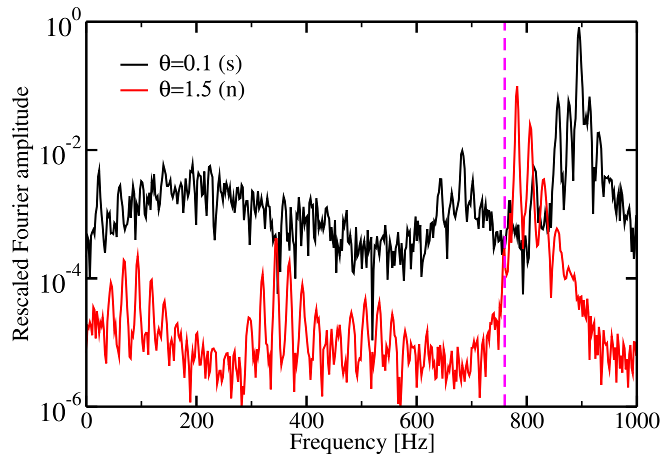

FIG. 3: Fourier transform of the velocity for a superfluid model near the polar axis ($\theta = 0.1$; black line), and for a normal fluid model near the equator ($\theta = 1.5$; red line). The dashed magenta line indicates the frequency $f = 760$ Hz of the $n = 1$, $l = 2$ crustal shear mode that was used as initial perturbation.

The $n = 1$ crustal shear modes propagate in radial direction. Because the magnetic field lines are almost orthogonal to this direction close to the equator, the coupling to the core is very weak in that region. This explains the structure of the QPO for the normal fluid case, together with the fact that magneto-elastic oscillations in the core are strongly reflected at the core-crust interface, which does not allow for a resonance between crust and core oscillations. At stronger magnetic fields, the Alfvén character of the magneto-elastic oscillations dominates, before the jump in propagation speed at the core-crust interface disappears. In contrast, in the superfluid case the strongest QPO at $f \sim 893$ Hz has its maximum close to the polar axis. Here, the shear terms dominate in the crust, and a higher magneto-elastic overtone in the core can enter in resonance.

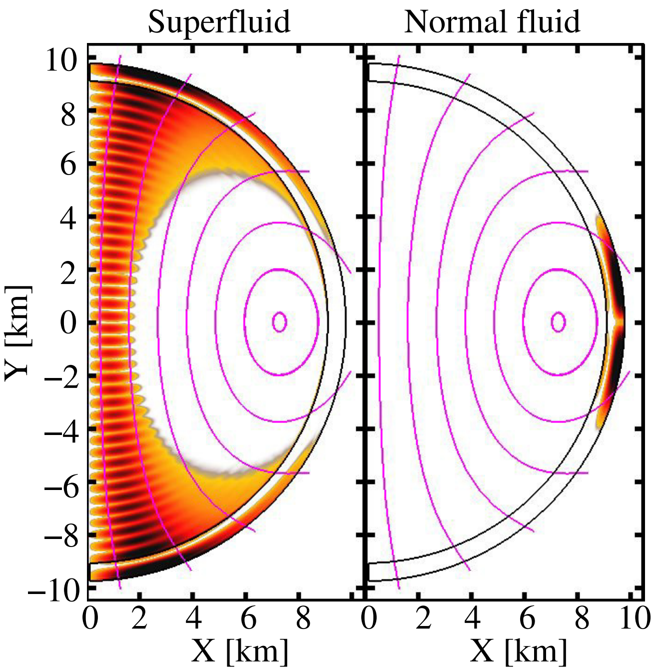

FIG. 4: Spatial distributions of the Fourier amplitudes of the velocity at the peak frequencies in Fig. 3. Left panel: QPO at $f \sim 893\,$Hz and $B = 5.4 \times 10^{14}\,$G for decoupled superfluid neutrons. Right panel: strongest shear dominated magneto-elastic ($n = 1$) QPO in the crust at $f \sim 782\,$Hz and $B = 10^{15}\,$G for a normal fluid core. Magnetic field lines are shown by magenta lines.

Overall, our results allow for a better understanding of the observed frequencies in SGR giant flares. The inclusion of superfluidity seems to be a key ingredient which helps in several ways: Firstly, the observed high frequency QPOs can be explained as global magneto-elastic QPOs resulting from a resonance between the crust and a high (~ 40) magneto-elastic overtone. This is only possible if there are superfluid neutrons in the core. There still may exist oscillations at frequencies above $500\,$Hz in models with normal fluid cores, but since these QPOs are limited to a region close to the equator they can only affect a small region of the magnetosphere close to the star. This makes it difficult to explain why QPOs are observed at different rotational phases [43]. Secondly, the phase of the magneto-elastic QPOs becomes constant for magnetic fields between several 10^{14}G to several $10^{15}\,$G. Due to the absence of phase mixing we expect that these QPOs are longer lived than magneto-elastic QPOs of normal fluid cores. We plan to investigate this in forthcoming work. Thirdly, the necessary magnetic field to match the low frequency QPOs $f \sim 30\,$Hz decreases by a factor of $\sqrt{1/X_p}$ which reduces our previous estimates $B \sim 1-4 \times 10^{15}\,$G [13] to $B \sim 2 \times 10^{14} - 10^{15}\,$G, in good agreement with [17, 31] and current spin down estimates for magnetars showing giant flares ($6 \times 10^{14} \lesssim B \lesssim 2.1 \times 10^{15}\,$G). A more realistic treatment of the entrainment is likely to further decrease our magnetic field estimates slightly [32].

These results do not only indicate the presence of a superfluid phase of neutrons in the core of SGRs, but they may also constrain the EoS of the crust significantly. The high frequency QPO and the threshold for the outbreak of the low frequency QPOs [13] give independent limits on the shear modulus of the crust, and hence on the EoS. We plan to investigate this in detail in forthcoming work. For the first time, our magnetar model that includes the effects of the crust, the magnetic field, and superfluidity can accommodate simultaneously all types of observed QPO frequencies, low ($f < 150\,$Hz) and high ($f > 500\,$Hz), in the giant flares of SGRs. For a particular model with a surface magnetic field strength of $B \approx 1.4 \times 10^{15}\,$G we find low frequency oscillations at 21, 30, 43, 58, 70, 74, 84, 89, 98, 119, 129, 135, 149, and $162\,$Hz that are in broad agreement with the QPOs observed in SGR 1806-20 at 18, 26, 30, 92, and $150\,$Hz.

More details of the theoretical framework and a careful analysis will be provided in forthcoming papers. The next major step towards a complete model for giant flare QPOs consists in finding a modulation mechanism of the emission in the magnetosphere.

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